

Assignment 1

Charge Density and Multiple Integrals

Definitions

charge density

- Line charge $\lambda = \frac{dq}{dl}$
- Surface charge $\sigma = \frac{dq}{dA}$
- Volume charge $\rho = \frac{dq}{dv}$

Questions

1. Find the total charge on the semicircle $x^2 + y^2 = R^2$, $z = 0$ for $0 \leq \varphi \leq \pi$ if $\lambda = \lambda_0$.
2. Find the total charge on the semicircle $x^2 + y^2 = R^2$, $z = 0$ for $0 \leq \varphi \leq \pi$ if $\lambda = \lambda_0 \sin^2(\varphi)$.
3. Find the total charge on the line $y=a$, $z = 0$ for $-a \leq x \leq a$ if $\lambda = \lambda_0 x^2 / a^2$.
4. Find the total charge on the hemisphere $x^2 + y^2 + z^2 = R^2$, $z \geq 0$ if $\sigma = \sigma_0 \cos(\theta) \sin^2(\varphi)$.
5. Find the total charge inside the hemisphere $x^2 + y^2 + z^2 = R^2$, $z \geq 0$ if $\rho = \rho_0 (r/R)^3 \cos(\theta) \sin^2(\varphi)$.
6. Find the total charge inside the cylinder $x^2 + y^2 = R^2$, $0 \leq z \leq L$ if $\rho = \rho_0 (z/L)^3 \sin^2(\varphi)$.
7. Find the total charge inside the pyramidal cone with a square base of side $2a$ ($-a \leq x \leq +a$ and $-a \leq y \leq +a$) and whose apex is that the point $(0,0,h)$ if the charge density is uniform.
8. Find the total charge inside the pyramidal cone with a square base of side $2a$ ($-a \leq x \leq +a$ and $-a \leq y \leq +a$) and whose apex is that the point $(0,0,h)$ if the charge density is given by $\rho = \rho_0 (z/h)$.