Worksheet – Motion of the Sun and Moon

Apparent Motion of the Sun

1. The Earth makes one complete orbit around the Sun in one year. From the point of view of someone on the Earth this makes the Sun appear to make one orbit around the Celestial Sphere in one year. We are going to approximate one year to be 360 days.
   a) How many degrees does the Sun appear to move around the Celestial Sphere in one day? (Round to the nearest integer value)
   
   \[
   \frac{360^\circ}{360 \text{ days}} = 1^\circ \text{ per day (approx)}
   \]

   b) By what angle does it move in one hour?
   
   \[
   \frac{1^\circ}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ hours}} = 0.0417^\circ \text{ per hour}
   \]

   c) Do you think you would notice this if you note the position of the Sun once an hour throughout the day?

   This is too small an angle to notice over the period of 1 hour.

Sidereal Day vs Synodic Day

2. The normal definition of the day (that is 24 hours) is the synodic day, that is from noon one day to noon the next day, noon being defined as the time that Sun crosses the meridian. This is caused by the rotation of the Earth, but the actual rotation period as measured relative to the stars (the sidereal day) is actually a little shorter
   a) We first need to convert the answer to question 1a from degrees to hours, and then to minutes. Remembering that everything moves across the sky at 15° per hour, convert you answer in degrees to an answer in hours.
   
   \[
   \frac{1^\circ}{15^\circ} \times \frac{1 \text{ hour}}{15} = \frac{1}{15} \text{ hours} = 0.0667 \text{ hours}
   \]

   b) What is this in minutes?
   
   \[
   \frac{1}{15} \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 4 \text{ minutes}
   \]

   c) What is the actual rotation period of the Earth, that is how long is the sidereal day?

   \[
   2 \text{ hours} – 4 \text{ minutes} = 23 \text{ h} 56 \text{ m}
   \]
Apparent Motion of the Moon

3. The Moon makes one complete orbit around the Earth in one month. We are going to approximate one month to be 30 days.
   a) How many degrees does the Moon appear to move in the sky in one day? (Round to the nearest integer value)

\[
\frac{360^\circ}{30 \text{ days}} = 12^\circ \text{ per day}
\]

b) By what angle does it move in one hour?

\[
\frac{12^\circ}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ hours}} = 0.5^\circ \text{ per hour}
\]

c) Do you think you would notice this if you note the position of the Moon once an hour throughout the night?

Yes, it is possible. You would probably need to find a reference object close to the Moon, such as a star or a planet, which you can easily find again an hour later. But if you can then the change in position of the Moon relative to this reference object is readily noticeable.

Sidereal Day vs Synodic Month

4. The normal definition of the month is the synodic month, that is from a full Moon one month to the next full Moon. This is related to the orbit of the Moon around the Earth, but the actual orbital period as measured relative to the stars (the sidereal month) is actually a little shorter.
   a) We first need to convert the answer to question 3a from degrees to hours, and then to minutes. Remembering that everything moves across the sky at 15° per hour, convert your answer in degrees to an answer in hours.

\[
12^\circ \times \frac{1 \text{ hour}}{15^\circ} = \frac{4}{5} \text{ hours} = 0.8 \text{ hours}
\]

b) What is this in minutes?

\[
0.8 \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 48 \text{ minutes}
\]
c) If the Moon has an azimuth of 290° at 10:00 tonight, at what time will it have an azimuth of 290° tomorrow?

\[10:00 + 0:48 = 10:48\]

d) At what time will it have an azimuth of 290° on this day next week?

\[10:00 + 7 \times 48 \text{ minutes} = 10:00 + 236 \text{ minutes} = 10:00 + 5:36 = 3:36 \text{ am}\]

e) What is the actual orbital period of the Moon? To answer this it is important to understand why the sidereal month and the synodic month are different. For now give the sidereal month the symbol \(T\). We are going to eventually solve for this number.

i. In a synodic month (30 days) have far does the Earth move in its orbit? (Hint see question 1.) To move from one full moon to the next the Moon has to orbit by 360° plus this angle.

In a synodic month the Earth moves 30° and so the Moon makes one complete orbit (360°) plus 30° or a total of 390°.

ii. Since we know that the Moon orbits 360° in a time \(T\), how long will it take to orbit through an angle of 360° plus this angle in the previous question?

\[
390° \times \frac{T \text{ days}}{360°} = 30 \text{ days}
\]

iii. Now set the answer to part ii equal to 30 days and solve for \(T\).

Solving for \(T\)

\[
T = 30 \text{ days} \times \frac{360°}{390°} = 27.7 \text{ days}
\]
Elongation and the phases of the Moon

First remind yourself of the definition of the elongation - the angle between the line connecting the Earth to the Sun and the line connecting the Earth to the Moon. This can be drawn either from the point of view of someone standing on the Earth, or from the point of view of someone outside the Solar System looking down on it.

If you point at the Sun, and the Moon is up to 180° to your left (that is further east) then the elongation is given the designation 'E'. If the Moon is to the right of you (that is further west) then the elongation is given the designation 'W'.

5. For each of the lines in the table, fill in either the phase of the Moon or the elongation. Don't forget to indicate whether a phase is waxing or waning.

<table>
<thead>
<tr>
<th>Elongation</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>New Moon</td>
</tr>
<tr>
<td>180° (E or W)</td>
<td>Full Moon</td>
</tr>
<tr>
<td>90° E</td>
<td>1st quarter</td>
</tr>
<tr>
<td>90° W</td>
<td>3rd quarter</td>
</tr>
<tr>
<td>135° W</td>
<td>Waning gibbous</td>
</tr>
<tr>
<td>21° E</td>
<td>Waxing crescent</td>
</tr>
<tr>
<td>85° W</td>
<td>Waning crescent</td>
</tr>
</tbody>
</table>

Time of day and the position of the Sun.

The time of day is determined only by the position of the Sun in the sky. Since the length of the daylight period depends on the date, and in a complicated way, we will simplify matter by only considering one of the two equinox days, for which the Sun rises due east of you at 6 am and sets due west of you at 6 pm. You can also imagine that this arc becomes a complete circle if you fill in the other half circle blow you.

6. Taking an arc across the sky (the ecliptic) with 0° marking the point where the Sun rises in the east and 180° where the Sun sets in the west, fill in the time or angle in the table below. Don't forget that the Sun moves are a rate of 15° per hour.
<table>
<thead>
<tr>
<th>Angle</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>6 am</td>
</tr>
<tr>
<td>90°</td>
<td>Noon</td>
</tr>
<tr>
<td>180°</td>
<td>6 pm</td>
</tr>
<tr>
<td>270°</td>
<td>midnight</td>
</tr>
<tr>
<td>45°</td>
<td>9 am</td>
</tr>
<tr>
<td>135°</td>
<td>3 pm</td>
</tr>
<tr>
<td>120°</td>
<td>2 pm</td>
</tr>
<tr>
<td>210°</td>
<td>8 pm</td>
</tr>
<tr>
<td>315°</td>
<td>3 am</td>
</tr>
</tbody>
</table>